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BUCKMINSTER FULLER: SYSTEMS THINKING, NATURE GEOMETRY AND TOPOLOGY – INSIGHT FROM HISTORY

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Abstract
This article presents the interdisciplinary achievements of the American architect and visionary Buckminster Fuller (1895–1983). They are considered with new attention from a historical perspective and in light of contemporary scientific research. Fuller's planet-friendly activity anticipated contemporary ecological concepts, treating Man and Nature as coexisting in unity. He introduced systems thinking and topology to architecture before digital technologies provided design tools for architects. He discovered that the tetrahedron is the basic building block of the simplest forms found in nature. In the 1940s he had already formulated an operational procedure for generating vector geometry and developed a computational apparatus that enabled the construction of spherical rod structures. The Climatron (1959) in the St. Louis Botanical Garden and the US Pavilion at Expo'67 in Montreal are presented, two spectacular structures that were ahead of their time. They are now valued not only for showing a way of controlling the environment through structural covering, but as a way of protecting life on Earth through technology. Concern for the environment and changes in science and culture that are evident at the beginning of the 21st century validate Fuller's efforts as a scientist, philosopher and architect, especially as the connections between his artifacts and the results of the latest scientific research have become obvious. His concept of eco-efficiency is currently being implemented and is an important alternative in architectural design in the era of global climate change.

Keywords: architecture; history; theory; systems thinking; topology; geometry of nature; geodesic structures

INTRODUCTION

Can Nature reveal the secrets of building its world on both the macro and micro scales? Will this allow humanity to survive successfully on planet Earth, and how? American architect and visionary Buckminster Fuller (1895–1983) dedicated his life to answering these questions. In the era of global climate change, Fuller's achievements are being analyzed and considered with renewed attention, particularly in terms of resource constraints and issues related to energy, water, and food security.

Fuller's planet-friendly interdisciplinary activity anticipated the current of thought in the second half of the 20th century, encompassing concepts that viewed humanity and Nature as a coexisting unity. He surrounded himself with other thinkers such as Theodor Roszak, George Bateson, and Fritjof Capra, who, in various fields (cultural history, anthropology, physics), propagated humanist and ecological views. Their worldview sought to overcome the Cartesian dualism of res extensa – res cogitans, which underpinned modern science, and viewed Nature as masses of matter subordinate to humanity.

Human activity has brought about the fulfillment of Fuller's predictions regarding the depletion of the planet's natural resources (such as timber, metal, coal, water, oil, etc.). There is hope still that with proper planning and foresight, designers and scientists could develop new ways to more efficiently utilize these precious natural resources to ensure the well-being of future generations. Fuller aimed to utilize design and science in a comprehensive manner to find solutions to these problems. He sought answers in the creations of Nature, whose forms are the most organized and efficient in every respect. He discovered that the tetrahedron is the basic building block of the simplest forms found in nature. In the 1940s, he formulated an operational procedure for generating vector geometry. Fuller called this geometry synergetic. Considering spherical and polyhedral forms as energy systems was unprecedented in the history of mathematics and geometry. By combining topology with vector geometry, it is now possible to explain, demonstrate, and transform behaviors found in Nature. Fuller presented his research results and philosophy in Sinergetics (1975) and Synergetics 2 (1979). The term is still used to describe phenomena ranging from politics to cellular automata, from economics to the theory of living systems.

Based on tetrahedral geometry, Fuller developed a computational apparatus in the early 1950s that enabled the construction of spherical rod structures. The Climatron (1959) at the St. Louis Botanical Garden and the US Pavilion at Expo'67 in Montreal are among the most spectacular structures, ahead of their time. Today, they are valued as an example of not only a designer's ambition to control the environment and the planet through structural covering, but also understood as his search for ways to protect life on Earth from the effects of ongoing climate change through technology.

The growing interest in creating new relationships between humans and the natural world (e.g., the EU's "Green Deal" program) encourages retrospection and analysis of primary scientific experiments, enabling the synthesis of information to construct a coherent description of events that occurred in the last century and are inspiring contemporary research. This broadens the cognitive scope of a specific segment of reality, which is the primary goal of scientific research, thus filling a gap in the theory and history of 20th-century architecture. In Poland, Fuller's achievements in the field of architecture are virtually unknown. The methodological approach adopted here combines descriptive-analytical techniques with logical reasoning.

The intention of this article is to present the interdisciplinary achievements of Buckminster Fuller from a historical perspective, as well as in light of contemporary scientific research. They remain inspiring and remain relevant, especially in the era of ongoing climate change on Earth. The need to adapt the built environment to the effects of these changes has opened a broad discourse on architectural design based on processes occurring in Nature which are being instrumentalized with the development of ICT. Increasingly refined IT design tools are emerging, the use of which is transforming the current understanding of the concept of imitation in architecture and art. These issues constitute a significant element of the international debate today on reconfiguring the concept of "Nature" in architectural discourse and the relationship between Humanity, Nature, and Technology.

1. FULLER'S CONCEPT OF GENERAL SYSTEMS

Fuller's systems thinking, as his biographers note, appeared in his early years of education (1902–1907), when he questioned the foundations of Euclidean geometry, noting its inconsistency with the real world. He disagreed that a chalk dot on a blackboard represented a mathematical point that contained nothing, or that a line could extend indefinitely. He considered this illogical [A.C. Edmonson 1992, p. 15]. These observations contributed to the formulation of a new systemic concept of worldview in the 1920s and 1930s. He first presented it in the manifesto 4D Time Lock (1928) and further developed it in Nine Chains to the Moon (1938). At the same time, in Vienna, biologist Ludwig von Bertalanffy (1901–1972) published his doctoral thesis "Kritische Theorie der Formbildung" (1928), which laid the foundation for the kinetic theory of open systems and, subsequently, general systems theory. Incidentally, in 1954, Bertalanffy founded the Society for General Systems Theory, whose goal was to further develop theoretical systems for application in many fields of knowledge. From this emerged a systems methodology, whose subject matter addresses inter- and multidisciplinary problems and a universal language for transdisciplinary communication common to various disciplines, and whose fundamental concept is the system [L. von Bertalanffy 1984]. Buckminster Fuller, on the other hand, worked independently.

At the end of the 19th century, particularly in the biological sciences, a crisis of mechanism and neopositivist philosophy emerged. Scientists protested against the deepening reductionism. However, only a few scientists were aware at the time that the reason for the limitations of thinking lay in respecting the Cartesian-Newtonian model. In his Discourse on Method, Descartes advocated breaking down every problem into simple elements, thoroughly examining these elements, and then assembling them into a well-understood, logical whole. Isaac Newton's classical mechanics became the universal theory for describing and characterizing the simplest phenomena. Propagated on this basis, reductionism adopted the principle of superposition, characterized by: i) mechanistic determinism, ii) independence of causes, iii) linearity of phenomena, and iv) analysis and mental synthesis. According to this principle, the properties of elements were assumed to be primary, and those of wholes were secondary, meaning that parts determined the whole. The abandonment of reductionism in science therefore required new tools and methods for studying phenomena and creations of Nature on both the macro and micro scale.

Questioning the Cartesian-Newtonian way of thinking, Fuller pursued many directions in his mathematical and geometrical experiments, and the key to these studies was the word system. For the ancient Greeks, σύστημα (systema) denoted a group of interrelated elements constituting a common entity. He therefore shared the view, attributed to Aristotle, that the whole is greater than the sum of its parts [H.W. Kruft 1994; K. Januszkiewicz 2010, p. 117]. The current scientific definition of a system describes a set of interrelated elements that interact with each other and was introduced by Ludwig von Bertalanffy [L. von Bertalanffy 1984, p. 69]. Fuller's intention was to provide new tools that could replace the questioned models. He believed that mathematical, physical, and geometric principles must be derived from experience. Then descriptions of real phenomena will be easy and understandable. He assumed that these principles could apply to both physical and non-physical structures, provided that one assumes that the Universe is the sum of conscious human experiences [B. Fuller, E.J. Applewhite 1975, item 301.10]. A system is understood by Fuller as a "conceivable entity". This entity allows us to distinguish between the internal and external parts of the system and constitutes a subdivision of the universe [B. Fuller, & E.J. Applewhite 1975, item 400.011]. For Fuller, a system is the first subdivision, the Universe. The universe is divided into six parts:

- geometric events of the universe occurring outside a given system;
- events of the universe occurring within the system;
- events of the universe occurring non-simultaneously and unrelated to events concerning the given system;
- non-simultaneous events of the universe resulting from events within the given system;
- all sets of geometric events constituting the given system as such;
- all events of the universe are synchronous or coincidental, which means that each set of system events is considered in a unique way. [B. Fuller, & E.J. Applewhite 1975, item 400.011].

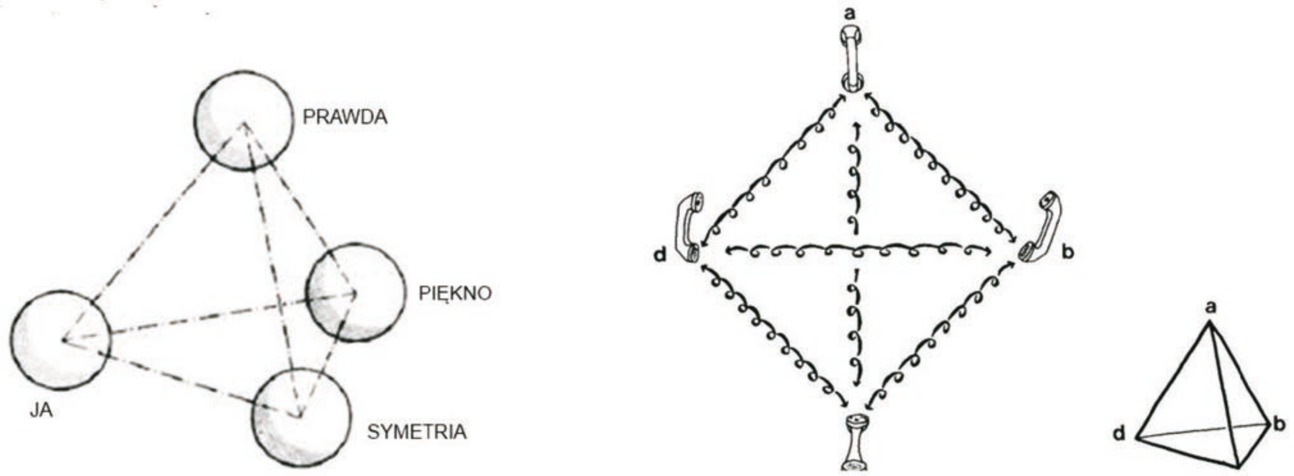
Fuller's concept of the Universe implies a complexity that is not static, changing moment by moment. To better understand the lack of simultaneity and the overlapping of events, Fuller compares it to a generational family, in which grandparents, parents, and children live, but these events do not overlap with all experiences [A.C. Edmonson, 1992, p. 11]. Comprehension means becoming aware of what is being understood. For comprehension to occur, information must be within the reach of human perception, and then it is perceived. To communicate means to inform oneself or others. This is not a scenario of a static whole.

To capture the essence of Nature, objectify it, and encapsulate it in geometric patterns and mathematical descriptions, Fuller assumed that the Universe is energy, and that every thought is woven into an incredibly complex web of interrelationships. It is therefore impossible to separate the physical from the nonphysical – together they constitute experience. Experience, however, is dynamic and composed of regenerative patterns of energy [A.C. Edmonson 1992, p. 11].

Fuller believes that the scientific principles that govern the interactions of event energies are timeless, just as eternal truths are, per se, nonphysical. Their line, therefore, is impossible to draw. Hence, the assumption that our awareness of event energies defines their occurrence is so crucial to Fuller's reasoning – humans cannot transcend the limitations of their understanding [A.C. Edmonson 1992, p. 11]. At the same time, he argued that the human mind has the unique ability to review many records of specific instances of experience stored in the brain. Occasionally, it discovers one of those rare scientific principles, which it generalizes after reviewing a set of experiences consistent in some respect. General principles are rules with no exceptions – from the simplest (such as the mechanical principle of the lever) to the more subtle (such as E = mc2, the formula for mass-energy equivalence) – taken together, they define the Universe [B. Fuller, & E.J. Applewhite 1975, p. xxvi]. He was convinced that if a reservoir of stored principles existed, then humanity's role was to discover and apply these truths. The mind is helpful (unlike the brain, which can only coordinate sensory input), and humanity is predestined to experience and sense reliable patterns, such as the law of universal gravitation ($F = GMm/R^2$). For Fuller, the discovery of gravity supported the validity of systems research, because the recognition of this invisible and unpredictable force resulted from the study of objects as separate entities. It was, as he called it, a fantastic leap beyond sensory information; this discovery made humanity aware of its contact with the greater Universe [A.C. Edmonson 1992, p. 12]. The undoubted influence of Einstein's theoretical work is evident here[1].

1.1. Minimal System

To present the system he calls minimal, Fuller instructs his listeners to imagine an element that, like the Greek point, is devoid of dimensions. Having two such elements, it is not yet possible to demarcate a field between them, because there are no boundaries. By adding a third point, you can only create a triangle with "nothing in the middle". In mathematics, three points lying non-linearly form a plane, and in a special case, a circle. When the fourth point is introduced, the situation changes. If the point does not lie on the plane determined by the three points, then the space is divided into two parts; one that is between the four points and the other that is left outside. This is how the minimal system is created [A.C. Edmonson 1992, p. 28]. For the Greeks, it was *a tetrahedron* (four sides). In order to define a sphere, four points are required that are not on the plane.



Tetrahedral Analysis of Plato's Triad

6 connections between 4 events define the tetrahedral system

Fig. 1. Buckminster Fuller, Minimal System and Energy System Geometry: Diagrams; source: [A.C. Edmonson 1992]

Fuller lamented that the Greek nomenclature corresponds only to the number of facets, which do not exist because they do not occupy space. They are not solids because they do not have continuous surfaces – they are only complex energy events or connections [A.L. Loeb 1976, p. 21]. "There are no geometric solids, no continuous surfaces (...) there are only complexes of energy events (and) relations" – claimed Buckminster Fuller [A.C. Edmonson 1992, p. 33]. Geometric solids, for Fuller, are only conceptual models that can only exist in the mind as a network of connections. These are conceptual wholes without size and exist outside of time. Size, however, is always a special case of experience [B. Fuller, E.J. & Applewhite 1975, item 515.14]. It belongs to a different category of parameters, like color, temperature, and durability.

Fuller treated geometry as the science of systems defined by relationships. Complex systems are, by necessity, polyhedral; they are finite aggregates of interconnected events that can be appropriately qualified. To understand the behavior of a system, these relationships can be diagrammed as polyhedral. This is a cognitive strategy that begins with the study of the whole and the known behaviors of its parts. It involves the progressive discovery of integral unknowns and the gradual understanding of the hierarchy of generalizable principles [B. Fuller & E.J. Applewhite 1975, item 152.00].

1.2. Nature's Coordination System

Can science today provide answers to the questions that puzzled Buckminster Fuller in the first half of the 20th century? Can we adequately explain the similarity of unrelated phenomena that differ in size and material? Do we know how to explain the magnificent orderliness of Nature? Does genetics already have an

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[1] In 1905, Einstein published groundbreaking work in Annalen der Physik, including the famous mass-energy equivalence formula ($E = mc^2$). In 1915, he announced the general theory of relativity, which stated that the effects of gravity and acceleration are equivalent.

indisputable answer to the canonical question of how a honeycomb, a seashell, or a virus is repeatedly modeled in the same form so as to manifest itself?

Based on his energetic system concept, Fuller discovered the fundamental principles that guide Nature in constructing its creations. These principles can be summarized as follows:

- Minimum energy consumption is the fundamental principle that guides Nature in building its structures. It is the energy needed to maintain the balance of external and internal forces [A.C. Edmonson 1992, p. 17].
- Spatial constraints. "Space is not merely a passive void, but has certain properties that impose certain constraints on any structure in order to be inhabited. These constraints are dependent on a specific interaction of forces, hence their nature is geometric" [A.C. Edmonson 1992, p. 17]. A simple example is the fact that representing space by only four polygons is effective when all are multiples of triangles. Spatial constraints are therefore not a function of material or size. "Natural" (e.g., shape) means what Nature permits [A.C. Edmonson 1992, p. 17].
- Its own self-organization. "Nature does not need to employ physics, biology, chemistry, and mathematics in its processes to decide how to produce a turnip or a virus. It merely emphasizes its own self-organization (...). The building is automatic. "Nature has only one department, the comprehensive coordinating system" [A.C. Edmonson 1992, p. 17].
- Self-building, how does this happen? Fuller assumes that it follows the path of least resistance, in accordance with the "minimum energy requirement", that is, systems automatically find a convenient organization that is, of necessity, the result of a balance of forces and spatial properties [A.C. Edmonson 1992, p. 17].

In the early 1930s, Fuller concluded that, given the nature of systems, his research should be geometric. Today, we know that geometry plays an important role in the morphogenesis of biological and computational forms. It not only describes the full development of form but also establishes boundaries and constraints that are local principles for self-organization during morphogenesis [N. Paszkowska-Kaczmarek 2023].

A coordinate system describes the shape and position of a body in space through a specific number, which is a component of that body. However, the position of that body can only be determined by relationships to other known locations or the original coordinate system. Mathematical functions locate points according to the adopted reference frames and determine their trajectories, which provides sufficient information to describe the entire system. Fuller points out that this information can be violated in two respects: shape and size. Based on observations, he assumes that "shape is purely spatial" [B. Fuller & E.J. Applewhite 1975, item 240.55]. It is easy to imagine identical shapes at different scales, such as an equilateral triangle, which by definition contains no indication of size. The lengths of the sides can be smaller or larger, but the angles must be 60 degrees. Shape is influenced only by the angle, and the angle does not depend on the lengths of the sides. The word triangle does not describe (without further modifications) the specificity of the shape, but its assumption – three related events without specifying lengths and angles.

Going beyond static mathematical concepts, Fuller believes that size, like other parameters, should always take into account "frequency". This word is intended to remind us of the role of time in all events that occur in the Universe. He introduces a distinction between time and duration. A real system is created by events, and they take time to happen [K. Januszkiewicz 2010, p. 117].

Nature's coordinating system is thus a geometry of economic relations that govern everything that is built. "If a geometry composed of a system of related vectors is discovered, it will represent a complete family of potential forces, a tendency toward proportional morphosis" [B. Fuller & E.J. Applewhite 1975, item 15.02.].

Fuller understood structure as a local and finite event that has a beginning and an end, because we cannot have the structure of the entire Universe. He defined it as a locally regenerative pattern of integration with the Universe. "Structure is a complex of events that integrate into a stabilized spatial pattern. This pattern is composed of actions, not things. It not only conducts energy but itself has stabilized energy. The ability to regenerate is an important feature because it signifies the transmission of Nature's energy. A structure must constantly regenerate itself in order to be sensed [B. Fuller & E.J. Applewhite 1975, item 606.01].

A crucial aspect of Nature's coordination system are the rules that ensure structure coherence. Fuller observed Nature and was certain that its construction involves a continuous transformation of energy. Atoms combine into clusters of "high frequency", separate periodically, and regroup to create new patterns, other substances. We interpret these patterns as solids or liquids because we are limited by our five senses. Patterns are completely independent of the medium, and therefore we receive information that they exist. Every chemical element is an integer formula. Fuller was convinced that Nature does not use the number when building, for example, bubbles or substances, but uses integers like H2O, not HπO [K. Januszkiewicz 2010, p. 117]. Could the number ϕ – (phi) therefore be the key to the Universe and ensuring its coherence? The solid “with impeccable capacity for transformation”, in Fuller’s view, was the polyhedron, whose logical models explain the behavior of Nature. According to ancient scholars, such as Aristotle, atoms of the elements were supposed to have perfect shapes, constituting regular polyhedra, still called Platonic solids. Fuller transformed these solids by truncating their vertices, as done by Archimedes and Johannes Kepler (1571–1630). Fuller’s polyhedra have been confirmed in the structure of natural crystals, and his geometric procedures are used in the design of new substances and materials for architecture [E.A. Lord et al. 2006].

2. GEOMETRY, VECTORS AND TOPOLOGY

For 40 years, Buckminster Fuller experimented (starting from scratch) in search of new geometric solutions, new methods, and new operating procedures. He ignored the geometry of curved surfaces and the spherical geometries of Gauss and Riemann (the opponents of Einstein’s theory of relativity). Like Kepler, he experimented with close-packing the sphere. He assumed that there must be some other mathematical model that needed to be quickly discovered. Mathematics, in turn, would provide a new “operating procedure”.

First, he built models starting from the center of a sphere. He drew lines connecting the centers of each packed sphere and developed a typology of vector relationships. These lines represented energy, its direction of flow, and time (Fig. 2). Instead of the geometric concept of a point, which is dimensionless, Fuller proposed an “energy event”. In this sense, any identifiable experience can be considered a point. It doesn’t matter whether it’s a single speck of dust or a plastic bag thrown in the street and seen from the top of the Empire State Building. It’s important that what constitutes such a point (particles, atoms, stars, planets) not be too diverse or too widely separated [B. Fuller & E.J. Applewhite 1975, pp. 505, 210].

Straight lines, however, possess only length, which cannot be demonstrated. Careful examination reveals that they are similar to waves or fragments of a trajectory; even the “line of sight” is a wave phenomenon, since “there are no straight lines in physics” [H. Kenner 1976, p. 23]. Only forces can have linear paths, which can be modeled as vectors. Hence, a continuous surface without thickness is replaced by a mesh of energetic events interconnected by vectors. According to Fuller, this understanding of geometry was intended to help better understand how Nature works. Today, such a mesh is a modeling tool in computer graphics [N. Paszkowska-Kaczmarek 2023].

In analyzing ball-packing models, Fuller focused on internal forces. He determined that the tetrahedron, not the cube, is the fundamental solid that builds the simplest forms that assimilate the energetic behaviors of spherical geometry. He discovered that the octahedron and icosahedron can also serve as the building blocks that most tightly fill space. For example, tetrahedra arranged one behind the other create a tetrahe-

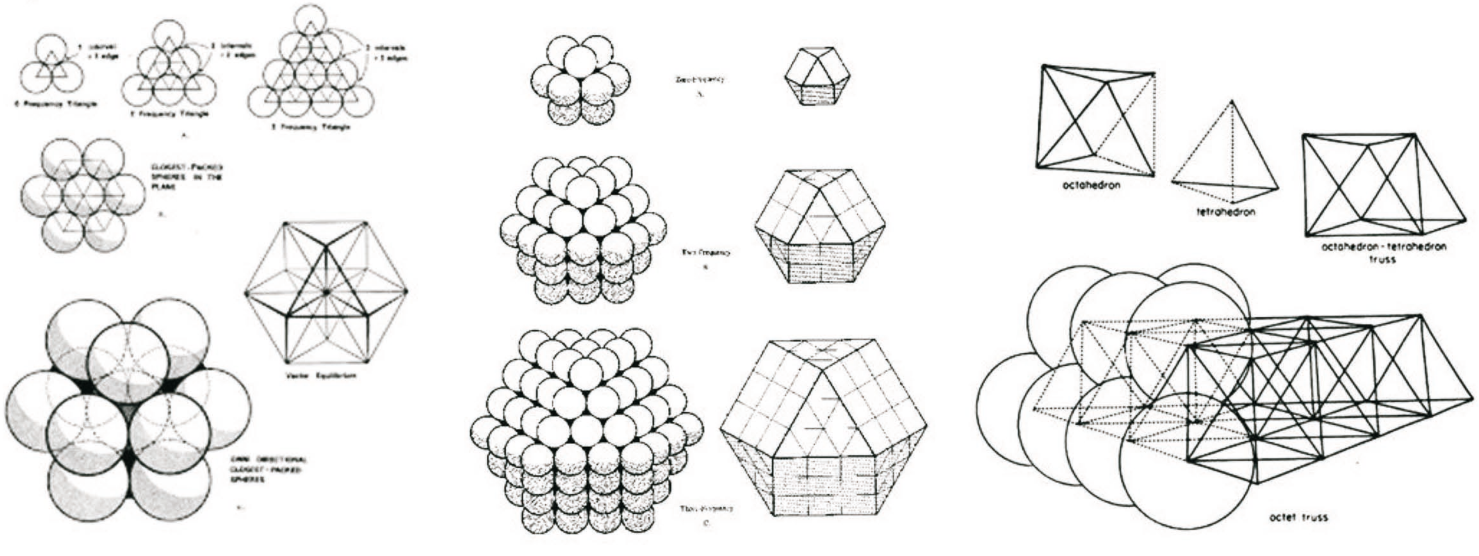
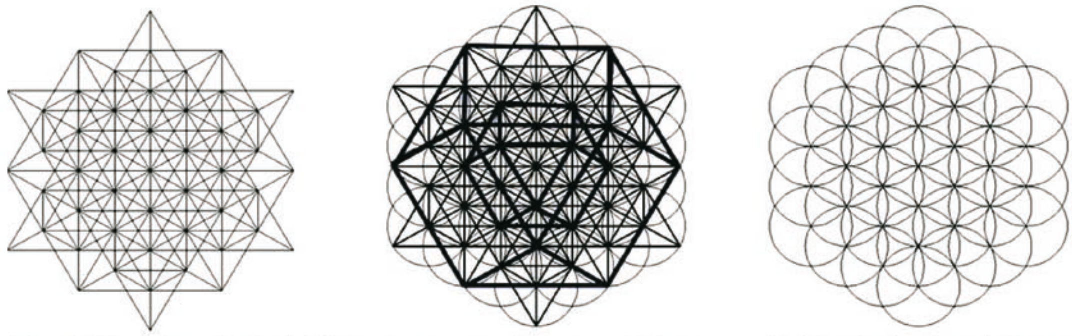


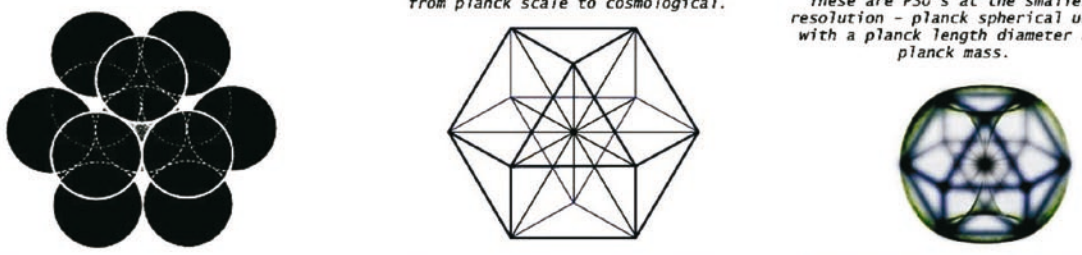
Fig. 2. Buckminster Fuller, Geometric experiments: close packing of a sphere; source: [A.C. Edmonson 1992]



64 Tetrahedron Grid - Othographic Projection. This is the seed space geometry that allows for all space dynamics we see, including expansion, collapse, and stillness.

Two octaves of vector equilibrium, centered/stillness at each octave, able to collapse at each octave due to jitterbugging/folding of cuboactehdron into octahedron. These octaves can be continually added, from planck scale to cosmological.

Each tetrahedron is a dual torus of implosion/expansion, a harmonic oscillator - resulting in a sphere around each. This is the literal holographic interference pattern of the cosmos - just as in holography. These are PSU's at the smallest resolution - planck spherical units with a planck length diameter and planck mass.



These are 'closest packed spheres' 12 around 1. This is the spherical analogue of the vector equilibrium - if each sphere were connected at their centerpoints. This is precisely why the VE is so stable.

Vector equilibrium / cubeoctahedron. Totally balanced due to edge and radial length being equivelant, however can also fold into itself for collapse. This is what the unperceivable vacuum energy's configuration usually is

Jitterbugging results in toroidal flow, collapse towards singularity

Fig. 3. Buckminster Fuller, Geometric Models of Energy Configurations: Vector Diagrams; source: [B. Fuller, E.J. Applewhite 1975]

lix – a helical structure that twists in both directions. Tetrahedrones and octahedrons packed together also fill space, and a 60-degree angle provides rigidity to such a structure in all directions. A 90-degree angle is merely an effect described as an octet truss [K. Januszkiewicz 2010, p. 120]. When the centers of closely packed spheres are connected, an Isotropic Vector Matrix (IVM) can be determined. Twelve spheres packed around one create the Vector Equilibrium, a state of equilibrium of multidirectional forces. This equilibrium can only be disturbed by the force of a potential explosion, equal to the forces acting in such a structure [B. Fuller, & E.J. Applewhite 1975, item 430.03]. Although some mathematical publications presented similar geometric drawings (tessellations of solids), they had never before been treated as vector diagrams [M. Ghyka, 1946]. A similar truss system had been previously described by Alexander G. Bell (1847–1922) for wooden structures, but Fuller was unaware of this at the time. He demonstrated the octet truss system in theory and practice as an extremely lightweight and efficient structural system [E.D. Harley 1982, p. 12].

In considering the geometric model describing energy states, Buckminster Fuller did not explicitly refer to the results of Max Planck's (1858–1947) energy research, which paved the way for quantum physics. Planck stated that energy cannot be radiated in arbitrary, continuous amounts, but only in the form of "portions" (quanta) of a specific value and frequency. Incidentally, Planck's monograph "The Principle of Conservation of Energy" (1887) has not yet fallen out of favor.

Experimenting with the geometry of closely packed spheres, Fuller assumed that each packed sphere is a sphere defining a field of interaction of a unit of energy, which can be configured and explained: "The geometric model of energy configuration in synergetics is developed from a symmetrical cluster of spheres, in which each sphere is a model of the energy field. In this cluster, all forces tend to self-coordinate, through momentary lateral displacements or pulsatile behavior, which occurs in both positive and negative asymmetric patterns, but they will never be in conflict with the eternal law of vector equilibrium." [B. Fuller & E.J. Applewhite 1975, item 205.01]. Fuller assumed that all matter is a specific state of energy, just as ice is a state of water. All that is needed is to discover the principle of this phenomenon and free the Earth from the burden of industry. Today, we speak of the chemistry and physics of quantum energy, i.e., the behavior of energy at the atomic level. Contemporary scientific research in quantum mechanics appears to confirm Fuller's hypotheses [M. Raymer 2024]. Incidentally, quantum energy is currently widely used in modern technology. Quantum computers, utilizing superposition phenomena, offer enormous computational capabilities, surpassing traditional computers in solving complex problems. Quantum energy is also revolutionizing medicine (diagnostic testing), telecommunications (data transmission), and the materials industry by designing new materials with unique properties. In the energy sector, it may lead to more efficient methods of energy storage and conversion.

In developing a mathematical "operational" procedure, Fuller draws on Euler's concept of graphs, which in 1736 gave rise to modern topology. Euler's equation, which establishes that the number of vertices (V) plus the number of facets (F) in any system equals the number of edges (E) plus two (V + F = E + 2). The number of vertices can therefore be precisely determined once the number of facets and edges is known. The constant number two, according to Fuller, represents the two poles through which the axis of rotation passes and must be derived from the number of all vertices for this equation to hold. All the numbers in this equation are integers.

According to the definition of a system, space is divided into an outer and an inner space – two cells. Could the constant 2 in Euler's equation represent such a division? It may be recalled that Euler's equation is a special case of the so-called Schläfli formula for any number of cells [F. Luo 2008]. That is, for multicellular structures, the number 2 in Euler's equation is replaced by C – the number of cells. Hence, V + F = E + C, even if C is greater than 2 [A.L. Loeb 1976, p. 11]. Such mathematics, based on integers, is easy to model or demonstrate and, as Fuller intended, will be understandable even by children under 10. It differs from conventional mathematics because it derives from experience – it is experimentally testable conceptual mathematics. By combining topology with vector geometry, one could easily explain, demonstrate, and process behaviors found in Nature.

Thanks to this approach, Fuller successfully redefined the geometry of the sphere. His experience during his military service in the US Navy from 1917 to 1920 undoubtedly played a significant role. As a navigator, he constantly determined new angles and curves to navigate through an ever-changing, fluid medium. This led Fuller, for the rest of his life, to consider terrestrial problems from a spherical perspective. He saw no flat surfaces anywhere. Everything was curved, from space to shape.

In the early 1940s, the results of geometric research allowed Fuller to formulate a complete operational procedure for generating vector geometry. He called this geometry synergetic. This introduced the concept of synergetics before German physicist Hermann Haken (born 1927) established synergetics in the 1980s as an interdisciplinary scientific field aimed at describing self-organizing processes[2].

The consideration of spherical and polyhedral forms as energy systems was unprecedented in the history of mathematics and geometry. Furthermore, Fuller's research demonstrated that self-similar elements occur in dynamic forms at every scale, from macro to micro. Today, the dynamic geometry of energy systems is a comprehensive discipline, broadly defined as the study of nonlinear spatial complexity in four dimen-

Table 1. Fuller's procedure for calculating the number of vertices, edges and facets based on Euler's equation

	V		F		E		2	Total
Tetrahedron	4	+	4	=	6	+	2	8
Octahedron	6	+	8	=	12	+	2	14
Cube	8	+	6	=	12	+	2	14
Icosahedron	12	+	20	=	30	+	2	32
Pentagonal dodecahedron	20	+	12	=	30	+	2	32

Source: [A.C. Edmonson 1992]

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[2] Hermann Haken (born 1927), inspired by laser theory, has been investigating the formation and self-organization of open systems since the early 1960s, exploring the formation and self-organization of structures beyond thermodynamic equilibrium. The term synergetics is derived from the Greek words sin (joint) and ergos (action). See: H. Haken, *Synergetics. Introduction and Advanced Topics*, Springer, 1983.

sions. This fourth dimension is revealed by organic and inorganic systems undergoing multidirectional transformations over time. This is demonstrated by molecular interactions and crystalline growth patterns, fractal structures visible in coastlines and tree branches, and in representations of DNA self-replication. Synergetic geometry in its mobile and ephemeral form can be demonstrated using the digital Java-Applet Geometries.

Fuller's geometric explorations align him with artists and scientists such as Pythagoras, Archimedes, Albrecht Dürer, Luca Pacioli, Leonardo da Vinci, and Johannes Kepler. Working in the intersection of art and science, music and technology, they observed the architecture of the universe. Each of them made a mark in the history of geometric research on symmetry, harmony, and balance, as well as the aesthetics of the "perfect sphere" in harmony with many spheres. Four hundred years later, Fuller continued a similar process of experimental observation of structures in four dimensions. The essence of Fullerian design was to discover connections between nature and science, art and technology, mathematics and music, which was evident in his approach to architecture [K. Januszkiewicz 2010, p. 121].

3. GLOBAL GEODESY AND TETRAHEDRAL GEOMETRY OF FULLER STRUCTURES

Geodesy deals with determining the most economical relationships between two events [B. Fuller & E.J. Applewhite 1975, item. 702.01]. This definition refers directly to Fuller's great circle geometry[3], which,

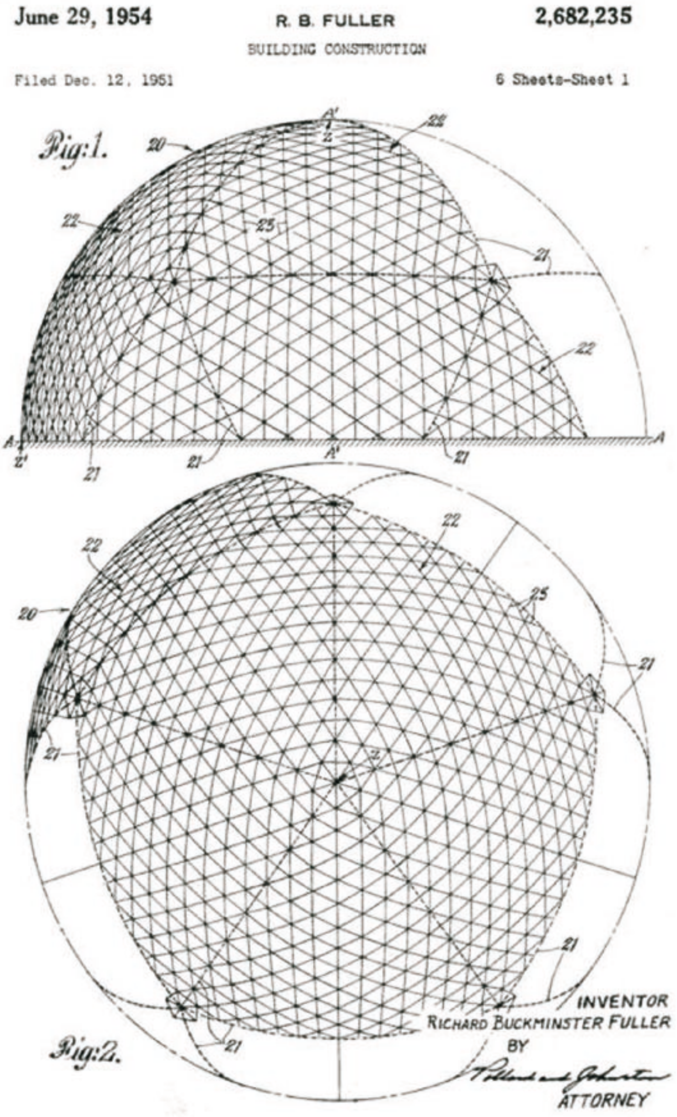

Fig. 4. Buckminster Fuller, spherical cover structure (half sphere), patent application, 1954; source: [T.T.K. Zung 2001]

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[3] Large circles result from the intersection of the sphere of the plane passing through the geometric center of the sphere.

in a spherical system, determine the shortest path between events, e.g. economic connections. However, the shortest distance between two points on the surface of a sphere is determined by a straight line (chord). The idea was to divide the surface of the sphere in such a way as to obtain a grid of flat equilateral triangles. Such components of the spherical system, appearing as flat, could belong to both the 'great circles' system and geodesy, as well as be considered in Fuller's tetrahedral geometry [K. Januszkiewicz 2010, p. 120].

Based on geodesics and tetrahedral geometry, Fuller developed a computational apparatus that enabled the construction of spherical rod structures. Although lightweight geodesic structures had already been designed by Walter Bauersfeld in Germany (Zeiss Planetarium, Jena, 1922), Fuller systematized the mathematical description of the *omnitrangulated* sphere. This system, called geodesic (Fuller's patent, 1954), is fundamental to structures based on the *icosahedron* (20-sided polyhedron). It allows objects to be built in any size and density of divisions. This makes an immeasurable contribution to the design of lightweight geodesic architecture.

In 1947, Buckminster Fuller patented the first spherical structural solution and has since licensed over two hundred such structures. Working with architects and engineering firms, he designed over 50 geodesic structures, including stadium roofs, exhibition pavilions, conservatories and other multi-space facilities (e.g. Multikino with F. L. Wright). He developed utopian visions of the widespread use of geodesic structures in various climatic and environmental conditions.

Buckminster Fuller built his first spherical structure in 1952 with students from Cornell University. The structure had a diameter of 6.4 metres and was named Geoscope. A model of this structure with a diameter of 12.8 metres was presented in 1954 at the 10th Milan Triennale, where it won the Grand Prize. In the same year, Fuller completed his first commercial project for the Ford Motor Company in Dearborn, Michigan – a dome with a diameter of 30 m. It weighed only 4.25 tonnes, as it was made of aluminium rods in a self-supporting tetrahedral system and covered with transparent polyester. From then on, interest in commercial spherical large-space coverings grew in the USA. At the end of the 1950s, the largest geodesic dome (116 m in diameter) was commissioned by the Union Tank Company in Baton Rouge (Louisiana) and a smaller one (108 m in diameter) in Wood River (Illinois). The military was interested in Fuller's lightweight structures for use in radar installations and for logistics tasks.

3.1. Cilimatron in St. Louis

In 1959, based on a system patented by Fuller just five years earlier, the world's first fully climate-controlled facility was completed at the St. Louis Botanical Garden (Fig. 5). Engineers from Murphy and Mackey St. Louis Engineers Synergetics, Inc. were also involved in the design and construction of the dome. The spherical (half-sphere) roof, 42 m in diameter and 21 m high, was designed based on the geometric and mathematical procedures formulated by Fuller (Fig. 5). It was made of tubular aluminium rods and Perspex panels. An innovative feature was the placement of these panels between the structural rods so that the surface of the sphere maintained geometric continuity. It is the lightest structure built to date, covering 320 m^2 of garden space. In addition to 400 species of tropical plants, there is also a classicist pavilion, a relic of the former city garden. Engineer Frits W. Went, a climate control specialist, used devices that allowed the climate of the Polynesian rainforests to be simulated. Cilimatron was honoured with the Reynolds Memorial Award of $25,000 for its pioneering use of aluminium in architecture [K. Januszkiewicz 2010, p. 122]. In 1976, the building was included in the list of the 100 most important architectural achievements in the history of the United States.

The interior of the Climatron is designed to resemble a tropical rainforest, emphasizing its diversity and ecology. Visitors immediately experience the tropics: dense green foliage, a small traditional hut, sparkling waterfalls, rocky cliffs, a river aquarium with exotic fish, and a platform from which to view the treetops and accompanying plants. Over 2,800 plants grow in the Climatron, including 1,400 different tropical species. These include banana trees, cocoa and coffee trees, many wild plants, orchids and exotic, rare plants such as the double coconut, which produces the largest seeds in the plant kingdom. Beneath the dome is a preexisting neoclassical stone pavilion. It is a spectacular display of four tropical plant ecosystems and climates, from the Amazon to Hawaii and Java to India, in a single multi-space interior. A set of 24 spotlights, rotating at night in five-minute cycles, simulates midday light on one side of the dome and moonlight on the other. The lush, green environment of a tropical rainforest is maintained by a computerized climate control system. The temperature inside ranges from 18°C at night to 29°C during the day. The average humidity is 85 per cent. The plants are irrigated with lukewarm water purified by reverse osmosis [Missouri Botanical Garden, n.d.].

The Climatron provides visitors with a climate-deterministic view of the tropics, subconsciously suggesting that any region of the planet can be artificially

Fig. 5. Buckminster Fuller, Climatron. Geodesic Dome, Missouri Botanical Garden, St. Louis, USA, 1959; source: [N. Paszkowska-Kaczmarek 2022]

recreated, regardless of its climatic conditions. In the final interior design (1960), the original three tropical zones were expanded to four: in one sector of the Climatron, both day and night temperatures are high, and a steamy Amazon jungle has been planted, complete with swamps and ponds. 'Little Hawaii' occupies another sector, with an oceanic climate, cool days and warm nights. Yet another sector has climatic conditions similar to those in the drier tropics and is used mainly for the presentation of cultivated tropical plants under the name 'India'. The remaining sector, with cool temperatures during the day and night, has the mountain climate of Java, where dense forests are covered in mist [R. Kanafani 2024, pp. 31–32].

The division of the four zones was managed by a sophisticated system controlled by a central computer, the Honeywell Supervisory Data Centre, which monitored and regulated the internal conditions of the Climatron. The Honeywell system, capable of completely replacing the air inside the dome in just two minutes, was connected to sensors and thermostats located throughout the structure to detect fluctuations in temperature and humidity. Using feedback, the system regulated the climatic conditions of each zone, providing the right environment for the growth of different plant species. The central computer was displayed in a glass case at the entrance.

The orangery was closed in 1988 for extensive renovation and reopened in 1990. Among other things, the damaged Plexiglas covering was replaced with new thermally reinforced glass panes, with Saflex (a plastic manufactured by Bayer Company) used as the interlayer. A total of 2,425 panes bent in two directions were used. To reduce costs, they were covered with low-emissivity film on the inside of the building. This coating helps to reduce heating costs by retaining the solar energy collected during the day for use at night. The new glass panel mounting system is rigid and has integral gutters to drain condensation [Missouri Botanical Garden, n.d.].

The Climatron was conceived both as an exhibition of Nature's creative possibilities and of Man's technological achievements based on structural patterns found in its creations. The metaphysical dimensions of this building remain largely unconscious, as does Fuller's quest to embody in material form what he called 'generalized principles' that refer to the immaterial structure of the Universe [R. Kanafani 2024].

In the era of global climate change, the Climatron should be interpreted more broadly. No longer just as an attempt to present these metaphysical principles combined with the ambition to control the environment and the planet through covering, but as the first demonstration of the possibility of protecting life on the planet from the effects of progressive climate change. By designing the Climatron, Buckminster Fuller initiated a new approach that would become relevant at the beginning of the 21st century, when buildings are treated as 'environmental valves' regulating the transmission of energy, light, air, moisture and information between the interior and exterior of the building [K. Januszkiewicz & N. Paszkowska 2016, p. 516].

3.2. The United States Pavilion at Expo'67 in Montreal

Fuller's domes gained widespread recognition for their technical, political and symbolic significance in representing American ingenuity on the world stage. They reflected the spirit of the era of the first space flights and intensified scientific research into space. In 1961, man saw Earth from a height of over 100 km (Earth's orbit) for the first time. The gates to the mythical Universe, the source of all perfection, opened. The captivating beauty of the image of planet Earth prompted reflection on its future.

The USA Pavilion at Expo'67 in Montreal is Fuller's most famous work, created in collaboration with Shoji Sadao and Geometrics Inc. The structure dominated the world exhibition grounds organized under the slogan 'Man and His World' as if it were a new planet. Fuller imagined that it would be the most 'controlled environment', and that its elastic, transparent and porous skin would provide visual contact with the world and daily modulation of the interior microclimate. Fuller was interested in how a surface could mimic the sensitivity and porosity of human skin, transmit light and act as an animated intelligent surface [T.M. Rohan 2003, p. 52].

The lightweight, spherical US Pavilion (3/4 sphere; Fig. 6a) with a diameter of 76.5 m and a height of 61 m is a homogeneous spatial structure made of ste-

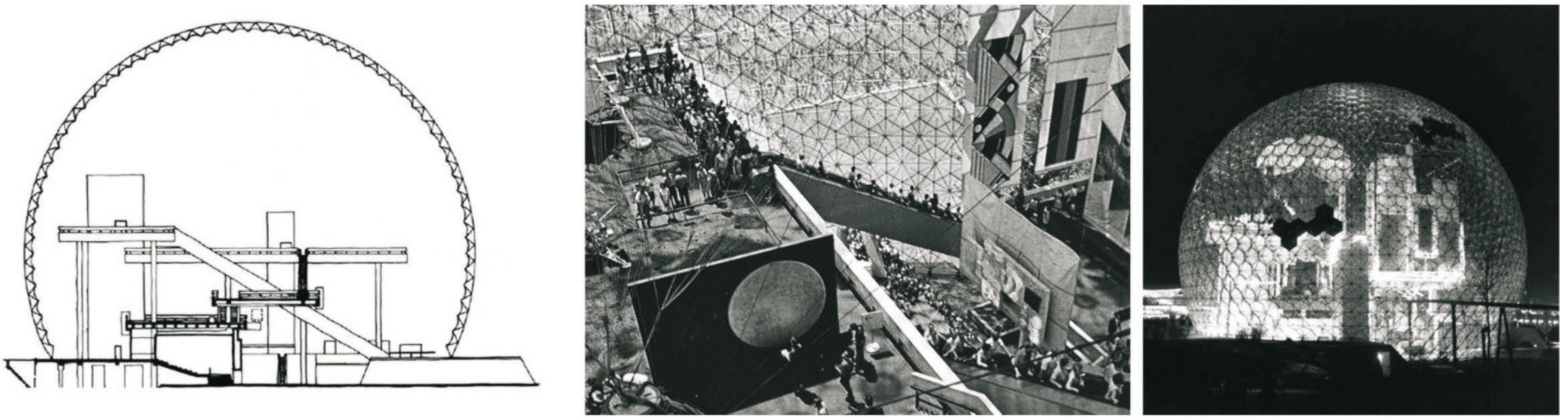

Fig. 6. Buckminster Fuller, Shoji Sadao, USA Pavilion, Expo'67, Montreal 1964, a) cross-section through the dome and exhibition platforms, b) view of the interior of the Pavilion c) night view of the Pavilion with the sun protection system; source: [J. Massey 2010]

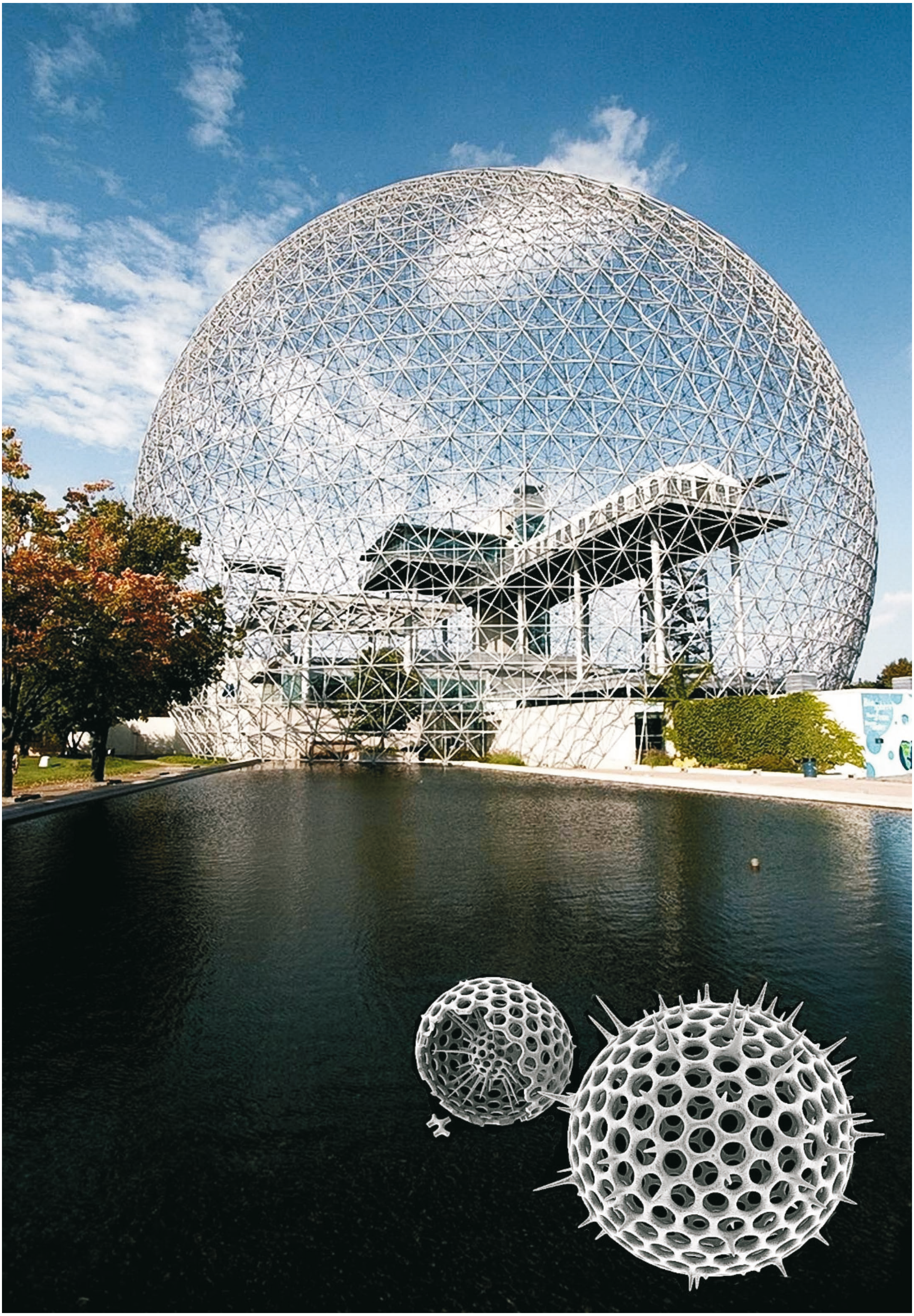

Fig. 7. Buckminster Fuller, USA Pavilion, Expo’64, Montreal, 1964 and radiolaria (1:770 enlargement); source: [N. Paszkowska-Kaczmarek 2022]

el rods. It was developed based on Fuller's operational procedure for generating vector geometry, which was based on the tetrahedron. Hence, the structure shows a triangular tessellation on the outside (Fig. 7) and a hexagonal tessellation on the inside (Fig. 6b). These hexagons allowed for the introduction of 1,900 convex acrylic panels on one side and a system of sunshades on the other (inside the building).

Fuller and Sadao proposed that the blinds be connected to a central computer and reset six times a day according to the movement of the sun, but this ambitious system was never implemented. Instead, mechanized triangular blinds were used, attached to about one-third of the inner surface of the dome (Fig. 6c). Controlled automatically by light sensors, the blinds adjusted to changing solar conditions. By combining the blinds with a thermostat controlled by conventional air conditioning equipment, automatic and cybernetic systems for maintaining a constant temperature were demonstrated, minimizing the consumption of fossil fuels. This technological solution mimicked the way the body regulates itself homeostatically to maintain a stable internal cell temperature, which is essential for its survival [J. Massey 2010, p. 464].

Fuller's vision was fully realized in 1995 when the building was equipped with new temperature and humidity control systems based on modern technology. In 1990, fourteen years after the fire that destroyed the pavilion's acrylic panels in 1976, it was purchased by Environment Canada. The pavilion was transformed into a unique interactive museum raising awareness and knowledge about the ecosystems of the Great Lakes and the St. Lawrence River. Today, the geothermal system and advanced technology allow for a reduction in energy consumption of 459 MWh (21% of annual consumption). The museum was opened in 1995 [K. Januszkiewicz 2010, p. 122].

In Montreal, the dreams of modernists came true; Moshe Safdie's modular Habitat was considered a prototype for mass-produced houses, and Frei Otto's suspended roof was the culmination of experiments conducted since the late 1930s. Megastructures such as the Gyotron entertainment center were the essence of the Cartesian-Newtonian mechanization of the structuralists of the time.

The USA Pavilion was not only a new structural solution, but also confirmed research into the world of nature at its most microscopic level – it was an imitation of it. Buckminster Fuller's structural sphere is a representation of 'microcosm' in the 'macrocosm'. An imitation of the structure of microorganisms described by D'Arcy W. Thompson in On Growth and Form, a book that caught Fuller's interest in the early 1950s.

The structure and shape of Fuller's sphere is confirmed by tiny marine creatures such as radiolarians, the structure of the cornea of the eye, and the structure of spherical viruses. The structure of these organisms is the result of a natural tendency to maintain internal cohesion and balance. The high density of a geodesic polyhedron also provides a model of a physical system that the human eye interprets as a sphere, e.g. a soap bubble. This is because the human eye cannot see individual molecules in a delicate, transparent soap film. They do not detect the forces of attraction between molecules running along the chords. Nevertheless, they exist, Fuller explained, and it is our duty to understand and communicate the truth about the Universe – to present tangible models of invisible phenomena.

The USA Pavilion was a confirmation of the order that Fuller found in Nature. He believed that Nature begins creating atoms and cells every minute and builds larger structures out of them. It was a fair, self-regulating, ideal system that humanity should see as a model for political and economic institutions. The sphere thus gained new symbolism, becoming not only a metaphor for the Planet and the World, but also for the global involvement of institutions and societies in international issues that are important to everyone. It is a manifestation of a new awareness in the age of science and technology, knowledge and experience. Fuller's concept of an ideal and non-hierarchical world was thus a harbinger of a new understanding of the broad spectrum of Man's relationship with Nature, Technology and Culture.

CONCLUSIONS

Buckminster Fuller (1895–1983) was one of the most revolutionary technological visionaries of the 20th century. Based on his own systems theory, Fuller recognized a universal design in the material world of Nature so that humanity could learn to build its artefacts – on a micro and macro scale. Developing on Euler's principles of topology, he developed a synergistic vector geometry for energy flow through a system that no one had previously invented or applied in architecture. He derived it from observations of the behavior of natural systems before science addressed the phenomena of emergence. Combining science with art, he sought solutions that would be beautiful and justified by optimal efficiency. He set new standards that can now be considered decisive in design aimed at limiting the effects of predatory economics on the planet. With the help of technology, humans have restructured and continue to restructure nature for the needs of modern society.

However, the world is currently undergoing a period of rapid change. These changes are ecological, social and economic.The demand for raw materials in the current capitalist economic paradigm has grown exponentially. At the same time, we are becoming increasingly aware not only of the limitations of the Earth's resources, but also of the destructive effects of their exploitation. The current imperative of environmental protection and sustainable development needs to be reconsidered today. This concerns in particular the issue of efficiency in terms of ecology.

Fuller demonstrated that constructing a shell with dense triangulation and icosahedral symmetry is the most efficient method of enclosing space (minimum material and maximum efficiency). Using the geometry he discovered through experimentation, he calculated, designed and built on a medium scale what Nature had done on a micro scale – a model of one of the strongest atomic bonds found in Nature. This was confirmed by science in 1985 when one of the greatest discoveries was made – the existence of the third form of carbon, C_{60}. This molecule looks like a football – a sphere made up of 12 regular pentagons and 20 hexagons, just like Fuller's geodesic domes from the 1950s and 1960s. The discovery of fullerenes is another step forward in research into the nanostructure of carbon, and fullerenes are a form of pure carbon. This sheds new light on the possibilities of action at the molecular level. It validates Fuller's intuitive belief that geodesic design plays a more significant role in the way Nature designs its creations than has been previously recognized. He believed that since the world consists of atoms and industry deals only with their useful applications, production should begin with the organization of atomic structures. This is how Buckminster Fuller imagined architecture being constructed in the early 1930s.

It seems appropriate to recall Fuller's achievements, especially since researchers at scientific centers in Europe and the USA are following the path he laid out. There is growing evidence that our planet has changed to such an extent that we are now living in a new historical era in which many key natural processes are dominated by human activity. Scientists agree that halting further degradation of the Earth requires rebuilding the relationship between humans, nature and the universe.

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